VISCOUS FLOW THEORY

LECTURE 6

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→ Two different methods are used to set up equations arising out of the physical principles

→ Differential element approach

→ Control volume approach

→ Differential element approach

→ A small fluid element is studied in terms of stresses acting on it

→ its responses to these stresses in terms of deformation rate

→ Control volume approach

→ Principles of conservation of mass and Newton's second law of motion are applied to a finite, fixed region in the flow field thru the Reynold's Transport Theorem

→ a third approach is used in continuum

→ Differential ~~equations~~ element approach leads to a system of differential eqns that describe the flow field

→ Control volume approach leads to an integral equations for the flow quantities

→ is more mathematically rigorous and doesn't assume the solution to be continuous before hand

→ very few techniques that can solve integral eqns are available

→ integral eqns form the starting point for a numerical solution using computational algorithms

Reynolds Transport Theorem

We need to describe the laws governing fluid motion using both system concepts (consider a given mass of the fluid) and control volume concepts (consider a given volume)

→ Velocity, acceleration, mass, temperature and momentum ~~are~~ a few common physical parameters

B — represents any fluid parameter

b — represents the amount of that parameter per unit mass

$$B = mb$$

if $B = m$, $b = 1$

$B = \frac{mV^2}{2}$, $b = V^2/2$

$B = m\vec{V}$, $b = \vec{V}$

B — extensive ~~parameter~~ property

b — intensive property

$$B_{sys} = \lim_{\delta V \to 0} \sum_i b_i (\rho_i \delta V_i)$$

$V \leftarrow$ volume

$$= \int_{sys} \rho b \, dV$$

amount of an extensive property can be determined by adding up the amount associated with each fluid particle in the system

→ Most of the laws governing fluid motion involve time rate of change of an extensive property of a fluid system

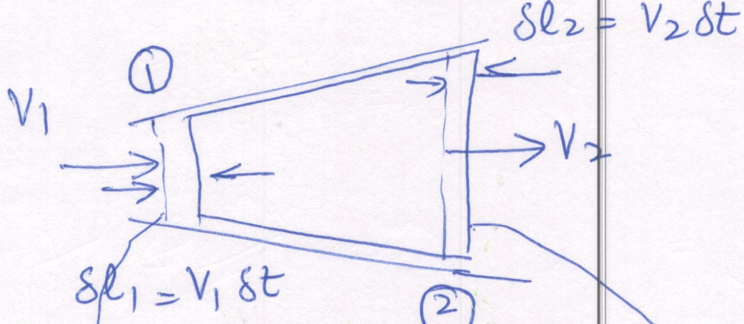
— rate at which the momentum of a system changes with time

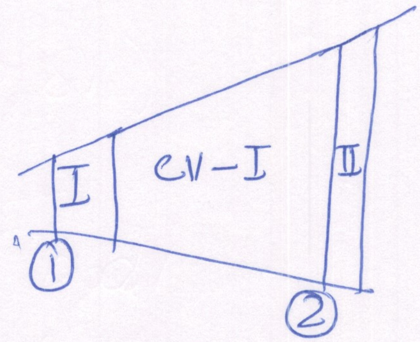
— rate at which mass of a system change with time & so on

$$\rightarrow \frac{dB_{sys}}{dt} = \frac{d \int_{sys} \rho b \, dV}{dt}$$

to formulate the laws into a CV approach we must obtain an expression for time rate of change of an extensive property within a control volume, B_{cv} not within a system

$$\int \rho b \, dV$$





CV surface and system boundary at time t

System boundary at time $t + \delta t$

→ Consider CV to be a stationary volume within the pipe or duct between section (1) & (2)

→ system − fluid occupying the CV at some initial time t

→ short time elapse δt

→ at $t + \delta t$, system has moved slightly to the right

→ fluid particles that coincided with section (2) of the CS at time have moved a distance $\delta l_2 = V_2 \delta t$ to right

→ fluid initially @ section (1) has moved a distance $\delta l_1 = V_1 \delta t$

V_1 V_2 are velocities at section 1 & 2 ... to the area & constant

(6)

at time t

$$sys = CV$$

at time $t + \delta t$

$$sys = CV - I + II$$

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if B is an extensive property of the system

$$B_{sys}(t) = B_{cv}(t)$$

at $t + \delta t$

$$B_{sys}(t+\delta t) = B_{cv}(t+\delta t) - B_{I}(t+\delta t) + B_{II}(t+\delta t)$$

Change in amount of B in the system in time interval δt divided by this time interval is given by

$$\frac{\delta B_{sys}}{\delta t} = \frac{B_{sys}(t+\delta t) - B_{sys}(t)}{\delta t}$$

$$= \frac{B_{cv}(t+\delta t) - B_{cv}(t)}{\delta t} - \frac{B_{I}(t+\delta t)}{\delta t}$$

$$+ B_{II}(t+\delta t)$$

lim $\delta t \to 0$

$$\frac{DB_{sys}}{Dt} = \lim_{\delta t \to 0} \frac{\delta B_{sys}}{\delta t} = \lim_{\delta t \to 0} \frac{B_{cv}(t+\delta t) - B_{cv}(t)}{\delta t}$$

$$- \lim_{\delta t \to 0} \frac{B_{I}(t+\delta t)}{\delta t} + \lim_{\delta t \to} \frac{B_{II}(t+\delta t)}{\delta t}$$

$$\lim_{\delta t \to 0} \frac{B_{cv}(t+\delta t) - B_{cv}(t)}{\delta t} = \frac{\partial B_{cv}}{\partial t}$$

$$= \frac{\partial \int_{cv} \rho b \, d\forall}{\partial t}$$

$$B_{II}(t+\delta t) = \rho_2 b_2 \delta \forall_{II} = \rho_2 b_2 A_2 V_2 \delta t$$

$$\delta \forall_{II} = A_2 \delta l_2 = A_2 V_2 \delta t$$

b_2 and ρ_2 are the const values of b and ρ across section (2)

$$\dot{B}_{out} = \lim_{\delta t \to 0} \frac{B_{II}(t+\delta t)}{\delta t} = \rho_2 A_2 V_2 b_2$$

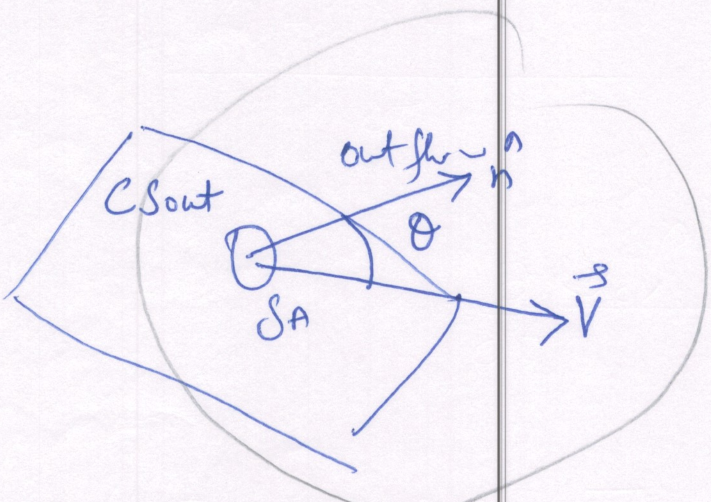
Similarly

$$\dot{B}_{in} = \lim_{\delta t \to 0} \frac{B_{I}(t+\delta t)}{\delta t} = \rho_1 A_1 V_1 b_1$$

Combine

$$\frac{DB_{sys}}{Dt} = \frac{\partial B_{cv}}{\partial t} + \dot{B}_{out} - \dot{B}_{in}$$

$$= \frac{\partial B_{cv}}{\partial t} + \rho_2 A_2 V_2 b_2 - \rho_2 A_1 V_1 b_1$$



$$\delta B = b\rho\delta\forall$$
$$= b\rho\,(V\cos\theta\,\delta t)\,\delta A$$

$$\delta\dot{B}_{out} = \lim_{\delta t \to 0} \frac{b\rho\delta\forall}{\delta t} = \rho b V\cos\theta\,\delta A$$

By integrating over the entire outflow portion CSout

$$\dot{B}_{out} = \int_{CSout} d\dot{B}_{out} = \int_{CSout} \rho b V\cos\theta\, dA$$

$$\dot{B} \quad \int \rho b \vec{V}\cdot\hat{n}\, dA$$

Similarly by considering the inflow portion

$$\dot{B}_{in} = -\int_{CS_{in}} \rho b V \cos\theta \, dA$$

$$= -\int_{CS_{in}} \rho b \vec{V} \cdot \hat{n} \, dA$$

$\dot{B}_{out} - \dot{B}_{in}$ = Net flux (flow rate) of parameter B across the entire control surface

$$\dot{B}_{out} - \dot{B}_{in} = \int_{CS_{out}} \rho b \vec{V} \cdot \hat{n} \, dA - \left(-\int_{CS_{in}} \rho b \vec{V} \cdot \hat{n} \, dA\right)$$

$$= \int_{CS} \rho b \vec{V} \cdot \hat{n} \, dA$$

Where integration is over the entire control surface

$$\left.\begin{aligned} I &= I_a + I_b + I_c + \ldots \\ II &= II_a + II_b + II_c + \ldots \end{aligned}\right\}$$ Grouping all inlets & exits

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$$\frac{DB_{sys}}{Dt} = \frac{\partial B_{cv}}{\partial t} + \int_{cs} \rho b \vec{V} \cdot \hat{n} \, dA$$

$$\boxed{\frac{DB_{sys}}{Dt} = \frac{\partial}{\partial t} \int_{cv} \rho b \, d\forall + \int_{cs} \rho b \vec{V} \cdot \hat{n} \, dA}$$

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